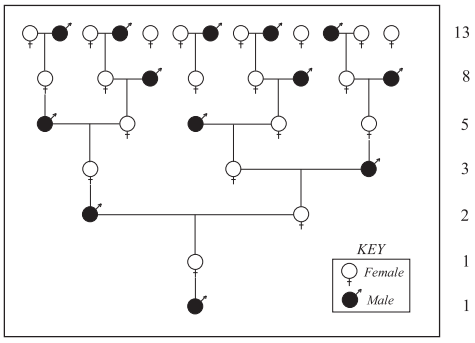
ACTIVITY 12.6 *Fibonacci's Sequence*

The Italian mathematician, *Leonardo de Pisa* (nicknamed *Fibonacci*) lived from about AD1170 to 1250. He devoted much of his time and effort to the study of the so-called *Fibonacci numbers*:

1, 1, 2, 3, 5, 8, 13, 21, 34, ...

This sequence appears frequently in the natural world, such as the pattern of reproduction in bees (shown opposite), the arrangement of leaves on stems, petals on flowers and spirals on cones.



Pattern of bee reproduction

1. Write down the next seven numbers in the sequence.

2. What is the general formula which generates the next number?

3. Consider the numbers:

   2, 3, 5, 8

   Multiply the two outside numbers and then the two inside numbers and note the difference.

   Try this with other sets of four consecutive Fibonacci numbers. *What do you notice?*

4. Square each of the first five consecutive Fibonacci numbers and add the results. Then multiply the fifth and sixth terms. *What do you notice?*

   Now square and add the first six numbers and multiply the 6th and 7th numbers. *Generalise your results.*

5. Consider the ratio of consecutive Fibonacci numbers.

   $$\frac{1}{1} = 1, \quad \frac{1}{2} = 0.5, \quad \frac{2}{3} = 0.\dot{6}, \quad \frac{3}{5} = 0.6, \quad \ldots$$

   To what number does this sequence tend? (This is called the *limit* of the sequence.)

   [Hint: Check the value of $\frac{-1+\sqrt{5}}{2}$.]

Extension

Investigate sequences obeying the Fibonacci relationship but with different starting values, e.g. replace (1, 1) by (1, 3).